Chapter 16

Recursion Table and Graph

You can input two formulas for any of the three following types of recursion, which you can then use to generate a table and draw graphs.

- General term of sequence $\{a_n\}$, made up of a_n and n
- Formulas for linear recursion between two terms, made up of a_{n+1}, a_n, and n
- Formulas for linear recursion between three terms, made up of a_{n+2}, a_{n+1}, a_n, and n

16-1 Before Using the Recursion Table and Graph Function

●To enter the RECUR Mode

On the Main Menu, select the **RECUR** icon and enter the RECUR Mode. This causes the Recursion Menu to appear.

Selected storage area

Press ▲ and ▼ to move.

- All recursion formulas that are stored in memory appear in the Recursion Menu.

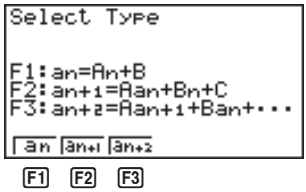
F1 (SEL) ..... Menu for control of table generation
F2 (DEL) ..... Recursion formula delete
F3 (TYPE) ..... Recursion formula type specification
F4 (n, a_n...) .... Menu for input of variable n and general terms a_n and b_n
F5 (RANG) .... Screen for setting of table range
F6 (TABL) ..... Recursion formula table generation

●To specify the recursion formula type

Before inputting a recursion formula, you must first specify its type.

1. In the Recursion Menu, press F3 (TYPE).

F3 (TYPE)



- In this display, "$a_n = An + B$" is the general term ($a_n = A \times n + B$) of $\{a_n\}$.

F1 (a_n) ..... General term of sequence $\{a_n\}$
F2 (a_{n+1}) ..... Linear recursion between two terms
F3 (a_{n+2}) ..... Linear recursion between three terms

2. Press the function key for the recursion formula type you want to set.

16-2 Inputting a Recursion Formula and Generating a Table

Example 1 **To input $a_{n+1} = 2a_n + 1$ and generate a table of values as the value of n change from 1 to 6**

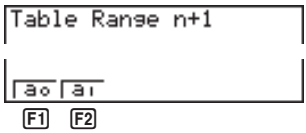
Make $a_1 = 1$.

1. Specify the recursion formula type as linear recursion between two terms and then input the formula.

[2] [F4] (n, a_n...)
[F2] (a_n) [+] [1]

2. Press [EXE] and then press [F5] (RANG) to display the table range setting screen.

[EXE] [F5] (RANG)



[F1] (a_0) ..... Value for a_0 (b_0)
[F2] (a_1) ..... Value for a_1 (b_1)

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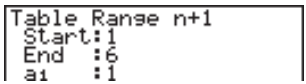
The table range settings specify the conditions that control the value of variable n in the recursion formula, and the initial term of the numeric value table. You should also specify a starting point for the pointer when drawing a convergence/divergence graph (WEB graph) for a formula for linear recursion between two terms.

Start ..... Starting value of variable n
End ..... Ending value of variable n
a_0, b_0 ..... Value of 0th term a_0/b_0 (a_1, b_1 .... Value of 1st term a_1/b_1)
a_nStr, b_nStr ..... Pointer starting point for convergence/divergence graph (WEB graph)

- The value of variable n increments by 1.

3. Specify the range of the table.

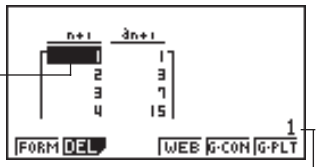
[F2] (a_1)
[1] [EXE] [6] [EXE] [1] [EXE]



4. Display the table of the recursion formula. At this time, a menu of table functions appears at the bottom of the screen.

EXIT F6 (TABL)

Currently selected cell (up to six digits)



Value in currently highlighted cell

- Displayed cell values show positive integers up to six digits, and negative integers up to five digits (one digit used for negative sign). Exponential display can use up to three significant digits.
- You can see the entire value assigned to a cell by using the cursor keys to move the highlighting to the cell whose value you want to view.
- You can also display the sums of the terms (Σa_n or Σb_n) by turning on Σ Display.

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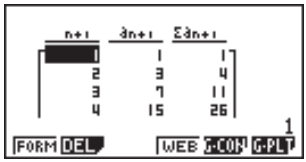


Example 2 **To input $a_{n+2} = a_{n+1} + a_n$ (Fibonacci series) and generate a table of values as the value of n change from 1 to 6**

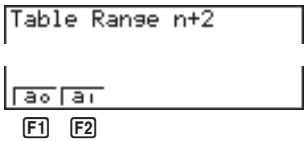
Make $a_1 = 1$ and $a_2 = 1$.

1. Specify the recursion formula type as linear recursion between three terms and then input the formula.

F3 (TYPE) F3 (a_{n+2})
F4 (n, a_n...)
F3 (a_{n+1}) + F2 (a_n)

2. Press EXE and then press F5 (RANG) to display the table range setting screen.

EXE F5 (RANG)



F1 (a_0) ..... Value for a_0 (b_0) and a_1 (b_1)
F2 (a_1) ..... Value for a_1 (b_1) and a_2 (b_2)

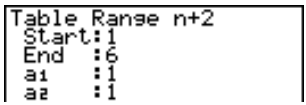
The table range settings specify the conditions that control the value of variable *n* in the recursion formula, and the initial term of the numeric value table.

Start ..... Starting value of variable *n*
End ..... Ending value of variable *n*
a_0, a_1, a_2 ..... Values of 0th term a_0/b_0, 1st term a_1/b_1, and 2nd term a_2/b_2.

- The value of variable *n* increments by 1.

3. Specify the range of the table.

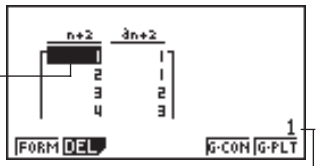
[F2] (a_1)
[1] [EXE] [6] [EXE] [1] [EXE] [1] [EXE]



4. Display the table of the recursion formula. At this time, a menu of table functions appears at the bottom of the screen.

[EXIT] [F6] (TABL)

Currently selected cell (up to six digits)



Value in currently highlighted cell

- There can be only one recursion table stored in memory at one time.
- Except for linear expression *n*, any of the following can be input for general term $\{a_n\}$ to generate a table: exponential expressions (such as $a_n = 2^n - 1$), fractional expressions (such as $a_n = (n + 1)/n$), irrational expressions (such as $a_n = \sqrt{n} - \sqrt{n-1}$), trigonometric expressions (such as $a_n = \sin 2n\pi$).
- Note the following points when specifying a table range.
  - If a negative value is specified as a start or end value, the calculator drops the negative sign. If a decimal value or fraction is specified, the unit uses only the integer part of the value.
  - If the value of a_0/b_0 (or a_1/b_1) is greater than the start value, the calculator makes the starting value of variable *x* the same as the value of a_0/b_0 (or a_1/b_1) before generating the table.
  - If the start value is greater than the end value, the calculator swaps the two values before generating the table.
  - If the start value is the same as the end value, the calculator generates a table using the start value of variable *x* only.
  - If the start value is very large, it may take a long time to generate a table for linear recursion between two terms and linear recursion between three terms.

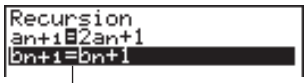
- Changing the angle unit setting while a table generated from a trigonometric expression is on the display does not cause the displayed values to change. To cause the values in the table to be updated using the new setting, display the table, press F1 (FORM), change the angle unit setting, and then press F6 (TABL).

●To specify the generation/non-generation status of a formula

Example **To specify generation of a table for recursion formula $a_{n+1} = 2a_n+1$ while there are two formulas stored**

▼ F1 (SEL)

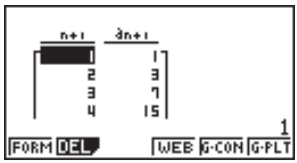
(Specifies non-generation status.)



Unhighlights this formula

F6 (TABL)

(Generates table.)



- To change the status of a recursion formula from non-generation to generation, select the formula and press F1 (SEL).

●To change the contents of a recursion formula

Changing the contents of a recursion formula causes the values in the table to be updated using the current table range settings.

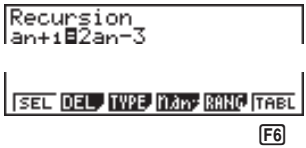
Example **To change $a_{n+1} = 2a_n+1$ to $a_{n+1} = 2a_n-3$**

▶

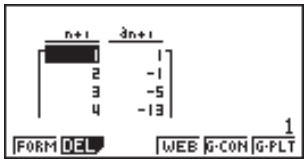
(Displays the cursor.)

▶ ▶ − 3 EXE

(Changes the formula contents.)



F6 (TABL)



●To delete a recursion formula

1. Display the Recursion Menu and then use ▲ and ▼ to highlight the formula you want to delete.

2. Press F2 (DEL).

3. Press F1 (YES) to delete the formula or F6 (NO) to abort the operation without deleting anything.

16-3 Editing Tables and Drawing Graphs

You get a choice of four options for editing tables and drawing graphs.

- Deletion of a recursion formula table
- Drawing of a connect type graph
- Drawing of a plot type graph
- Drawing of a graph and analysis of convergence/dievergence (WEB)

You can access these options from the function menu that appears at the bottom of the screen whenever a table is displayed.

F1 (FORM) .... Returns to Recursion Menu.
F2 (DEL) ..... Table delete
F4 (WEB) ..... Draws convergence/divergence graph (WEB graph).
F5 (G•CON) .. Draws connected type recursion graph.
F6 (G•PLT) .... Draws plot type recursion graph.

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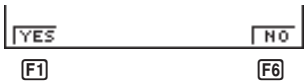
- The WEB item (F4) is available only when a table generated using a formula for linear recursion between two terms (a_{n+1} =, b_{n+1} =) is on the display.

●To delete a recursion table

1. Display the recursion table you want to delete and then press F2 (DEL).



2. Press F1 (YES) to delete the table or F6 (NO) to abort the operation without deleting anything.

■ Before Drawing a Graph for a Recursion Formula

You must first specify the following.

- Draw/non-draw status of for the recursion formula
- The type of data to be plotted

To specify the draw/non-draw status, display the Recursion Menu and then press F1 (SEL).

●To specify the draw/non-draw status of a formula

There are two options for the draw/non-draw status of a recursion formula graph.

- Draw the graph for the selected recursion formula only
- Overly the graphs for both recursion formulas

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To specify the draw/non-draw status, use same procedure as that for specifying generation/non-generation status.

●To specify the type of data to be plotted (Σ Display: On)

You can specify one of two types of data for plotting.

- a_n on the vertical axis, n on the horizontal axis
- Σa_n on the vertical axis, n on the horizontal axis

In the function menu that appears while a table is on the display, press F5 (G•CON) or F6 (G•PLT) to display the Plot Data Menu.

F1 (a_n) ..... a_n on the vertical axis, n on the horizontal axis

F6 (Σa_n) ..... Σa_n on the vertical axis, n on the horizontal axis

Example 1 **Draw a graph of $a_{n+1} = 2a_n + 1$ with a_n on the vertical axis and n on the horizontal axis, and with the points connected.**

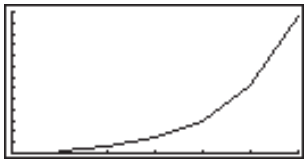
Set the following parameters in the View Window.

Xmin = 0	**Ymin = 0**
Xmax = 6	**Ymax = 65**
Xscale = 1	**Yscale = 5**

F6 (TABL) F5 (G•CON)
(Selects connected type.)

F1 (a_n)

(Draws graph with a_n on the vertical axis.)



Example 2 **Draw a graph of $a_{n+1} = 2a_n + 1$ with Σa_n on the vertical axis and n on the horizontal axis, and with the points unconnected.**

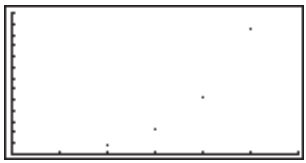
Use the same View Window parameters as those provided in Example 1.

F6 (TABL) F6 (G•PLT)

(Selects plot type.)

F6 (Σa_n)

(Draws graph with Σa_n on the vertical axis.)



- To input a different recursion formula after a graph is drawn, press SHIFT QUIT. This displays the Recursion Menu where you can input a new formula.

■ Drawing a Convergence/Divergence Graph (WEB graph)

With this feature, you can draw a graph of $a_{n+1} = f(a_n)$ where a_{n+1} and a_n are the terms of linear recursion between two terms, substituted respectively for y and x in the function $y = f(x)$. The resulting graph can then be viewed to determine whether or not the graph is convergent or divergent.

Example 1 **To determine whether or not the recursion formula $a_{n+1} = -3a_n^2 + 3a_n$ is convergent or divergent.**

Use the following table range.

Start = 0 **End = 6**
a_0 = 0.01 **a_n Str = 0.01**
b_0 = 0.11 **b_n Str = 0.11**

Use the following View Window parameters.

Xmin = 0 **Ymin = 0**
Xmax = 1 **Ymax = 1**
Xscale = 1 **Yscale = 1**

This example assumes that the following two recursion formulas are already stored in memory.

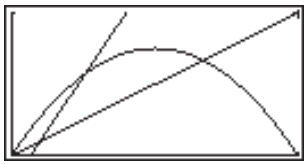
F6

1. Press F6 (TABL) to generate a table.

   F6(TABL)

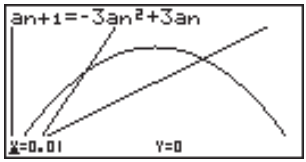
2. Press F4 (WEB) to draw the graph.

   F4(WEB)



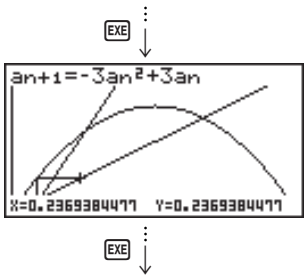
3. Press EXE, and the pointer appears at the pointer start point (a_nStr = 0.01).

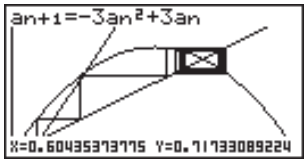
   EXE



- The Y value for the pointer start point is always 0.

4. Each press of EXE draws web-like lines on the display.





This graph indicates that recursion formula $a_{n+1} = -3a_n^2 + 3a_n$ is convergent.

Example 2 **To determine whether or not the recursion formula $b_{n+1} = 3b_n + 0.2$ is convergent or divergent.**

Use the following table range.

Start = 0 **End = 6**

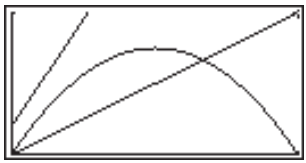
b_0 = 0.02 **b_n Str = 0.02**

Use the View Window parameters from Example 1.

F6

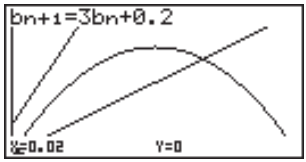
1. Press F6 (TABL) F4 (WEB) to draw the graph.

F6 (TABL)
F4 (WEB)



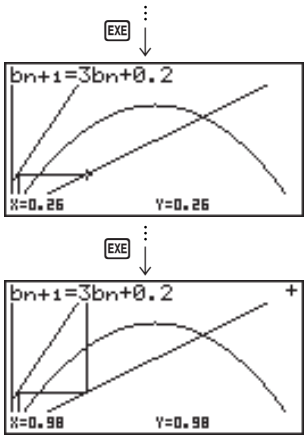
2. Press EXE and then either ▲ or ▼ to make the pointer appear at the pointer start point (b_nStr = 0.02).

EXE ▼ (or ▲)



- The Y value for the pointer start point is always 0.

3. Each press of [EXE] draws web-like lines on the display.



This graph indicates that recursion formula $b_{n+1} = 3b_n + 0.2$ is divergent.

- Inputting b_n or n for the expression a_{n+1}, or Inputting a_n or n for the expression b_{n+1} for linear recursion between two terms causes an error.

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●To draw a recursion formula graph using Dual Screen

Selecting "**T+G**" for the Dual Screen item of the Set Up Screen makes it possible to display both the graph and its numerical table of values.

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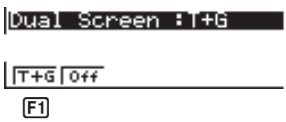
Example **To draw the graph of $a_{n+1} = 2a_n + 1$ from Example 1, displaying both the graph and its table**

[SHIFT] [SETUP]
▼ ▼ ▼ [F1] (T+G)
(Specifies T+G for Dual Screen.)



[EXIT] [F6] (TABL)
(Draws the graph and shows the table.)

[F6] (G•PLT)
(Draws plot type graph.)

- Pressing [SHIFT] [F6] (G↔T) causes the graph on the left side of the Dual Screen to fill the entire display. Note that you cannot use the sketch function while a graph is displayed using [SHIFT] [F6] (G↔T).