Chapter **14**

14

Implicit Function Graphs

You can graph any one of the following types of implicit functions using the calculator's built-in functions.

- Parabolic graph
- Circle graph
- Elliptical graph
- Hyperbolic graph

14-1 Before Graphing an Implicit Function

■ Entering the CONICS Mode

1. In the Main Menu, select the **CONICS** icon and enter the CONICS Mode. When you do, the following built in function menu appears on the screen.

```
Select Equation
X=A(Y-K)²+H      ⟵
X=AY²+BY+C       ⟵
Y=A(X-H)²+K      ⟱
```
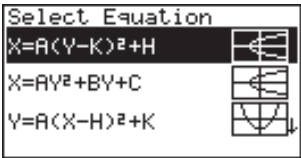
2. Use ⊙ and ⊙ to highlight the built-in function you want, and then press [EXE].

The following nine functions are built in.

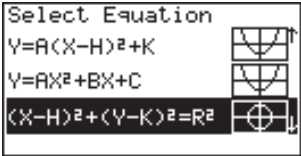| Graph Type	Function
Parabola	$X = A(Y - K)^2 + H$ $X = AY^2 + BY + C$ $Y = A(X - H)^2 + K$ $Y = AX^2 + BX + C$
Circle	$(X - H)^2 + (Y - K)^2 = R^2$ $AX^2 + AY^2 + BX + CY + D = 0$
Ellipse	$\dfrac{(X - H)^2}{A^2} + \dfrac{(Y - K)^2}{B^2} = 1$
Hyperbola	$\dfrac{(X - H)^2}{A^2} - \dfrac{(Y - K)^2}{B^2} = 1$ $\dfrac{(Y - K)^2}{A^2} - \dfrac{(X - H)^2}{B^2} = 1$

14-2   Graphing an Implicit Function

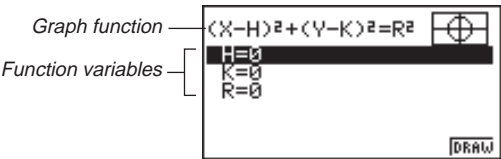**Example 1**   **To graph the circle $(X - 1)^2 + (Y - 1)^2 = 2^2$**

Use the following View Window parameters.

Xmin = –6.3	Ymin = –3.1
Xmax = 6.3	Ymax = 3.1
Xscale = 1	Yscale = 1

1. Select the function whose graph you want to draw.

   $\odot \odot \odot \odot$

   ```
   Select Equation
   Y=A(X-H)²+K
   Y=AX²+BX+C
   (X-H)²+(Y-K)²=R²
   ```

2. Press [EXE] and the variable input screen appears.

   [EXE]

   Graph function ——— (X-H)²+(Y-K)²=R²

   Function variables ———
   ```
   H=0
   K=0
   R=0
                              DRAW
   ```

   • The values that appear are the values currently assigned to each variable, which are general variables used by the calculator. If the values include an imaginary part, only the real part appears on the display.

3. Assign values to each variable.

   [1] [EXE]
   [1] [EXE]
   [2] [EXE]

   ```
   (X-H)²+(Y-K)²=R²
   H=1
   K=1
   R=2
                              DRAW
   ```
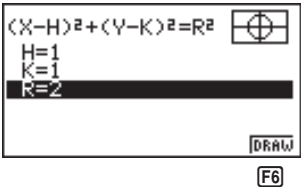
   • You can also use $\odot$ and $\odot$ to highlight a variable and then input a value.

   [F6]

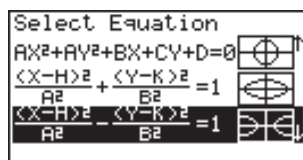4. Press [F6] (DRAW) to draw the graph.

   [F6] (DRAW)

P.155
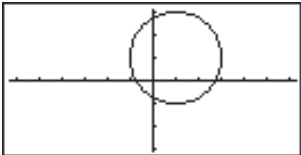
- Certain View Window parameters can make a circle graph come out looking like an ellipse. When this happens, you can use the graph correction function (SQR) to make corrections and produce a perfect circle.

Example 2 To graph the hyperbola $\dfrac{(X-3)^2}{2^2} - \dfrac{(Y-1)^2}{2^2} = 1$
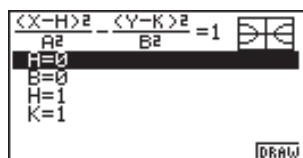
Use the following View Window parameters.

Xmin = –8	Ymin = –10
Xmax = 12	Ymax = 10
Xscale = 1	Yscale = 1

1. Select the function whose graph you want to draw.

   ⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙

   ```
   Select Equation
   AX²+AY²+BX+CY+D=0  ⊕
   (X-H)²  (Y-K)²
   ──── + ──── =1    ⊜
    A²      B²
   (X-H)²  (Y-K)²
   ──── - ──── =1    ⊃⊂
    A²      B²
   ```

2. Press [EXE] and the variable input screen appears.

   [EXE]

   ```
   (X-H)²  (Y-K)²
   ──── - ──── =1   ⊃⊂
    A²      B²
   A=0
   B=0
   H=1
   K=1
                    DRAW
   ```

3. Assign values to each variable.

   [2] [EXE]
   [2] [EXE]
   [3] [EXE]
   [1] [EXE]

   ```
   (X-H)²  (Y-K)²
   ──── - ──── =1   ⊃⊂
    A²      B²
   A=2
   B=2
   H=3
   K=1
                    DRAW
   ```
   [F6]
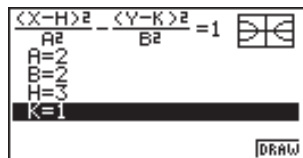
4. Press [F6] (DRAW) to draw the graph.

   [F6] (DRAW)
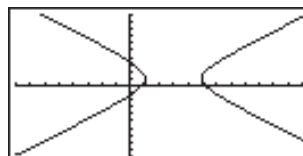
• A parabola is the locus of points equidistant from fixed line l and fixed point F not on the line. Fixed point F is the "focus," fixed line l is the "directrix," the horizontal line that passes through the focus directrix is the "axis of symmetry," the length of a straight line that intersects the parabola, passes through the locus, and is parallel to fixed line l is the "latus rectum," and point A where the parabola intersects the axis of symmetry is the "vertex."
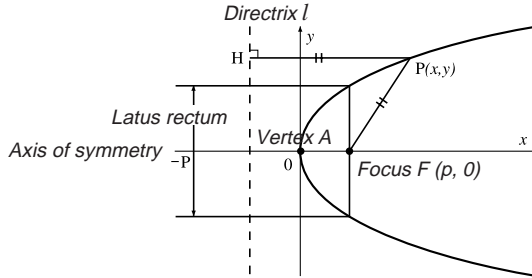
• An ellipse is the locus of points the sum of the distances of each of which from two fixed points F and F' is constant. Points F and F' are the "foci," points A, A', B, and B' where the ellipse intersects the x- and y-axes are the "vertexes," the x-coordinate values of vertexes A and A' are called x-intercepts, and the y-coordinate values of vertexes B and B' are called y-intercepts.
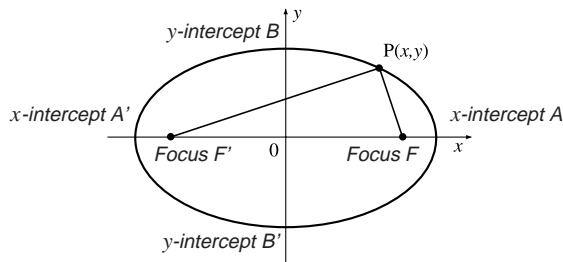
• A hyperbola is the locus of points related to two given points F and F' such that the difference in distances of each point from the two given points is constant. Points F and F' are the "foci," points A and A' where the hyperbola intersects the x-axis are the "vertexes," the x-coordinate values of vertexes A and A' are called x-intercepts, the y-coordinate values of vertexes A and A' are called y-intercepts, and straight lines i and i' , which get closer to the hyperbola as they move away from the foci are "asymptotes."

227
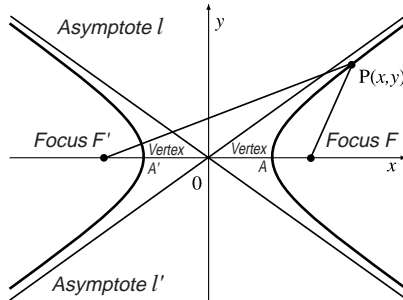
14-3   Implicit Function Graph Analysis

You can determine approximations of the following analytical results using implicit function graphs.

- Focus/vertex calculation
- Latus rectum calculation
- Center/radius calculation
- x-/y-intercept calculation
- Directrix/axis of symmetry drawing and analysis
- Asymptote drawing and analysis

After graphing an implicit function, press [F5] (G-Solv) to display the Graph Analysis Menu.

- Parabolic Graph Analysis

    [F1] (FOCS) .... Determines the focus.

    [F2] (SYM) .... Draws the axis of symmetry.

    [F3] (DIR) .... Draws the directrix.

    [F4] (VTX) .... Determines the vertex.
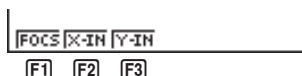
    [F5] (LEN) .... Determines the latus rectum.

FOCS	SYM	DIR	VTX	LEN
[F1]	[F2]	[F3]	[F4]	[F5]

- Circle Graph Analysis

    [F1] (CNTR) .... Determines the center.

    [F2] (RADS) .... Determines the radius.

CNTR	RADS
[F1]	[F2]

- Ellipse Graph Analysis

    [F1] (FOCS) .... Determines the focus.

    [F2] (X-IN) .... Determines the x-intercept.

    [F3] (Y-IN) .... Determines the y-intercept.

FOCS	X-IN	Y-IN
[F1]	[F2]	[F3]

- Hyperbolic Graph Analysis

    [F1] (FOCS) .... Determines the focus.

    [F2] (X-IN) .... Determines the x-intercept.

    [F3] (Y-IN) .... Determines the y-intercept.

    [F4] (VTX) .... Determines the vertex.

    [F5] (ASYM) .... Determines the asymptote.

FOCS	X-IN	Y-IN	VTX	ASYM
[F1]	[F2]	[F3]	[F4]	[F5]
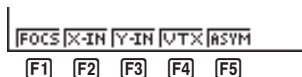
The following examples show how to use the above menus with various types of implicit function graphs.

● **To calculate the focus and vertex**

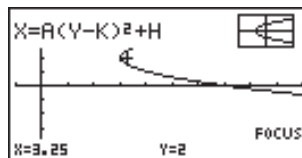**Example** **To determine the focus and vertex for the parabola**
X = (Y − 2)² + 3.

Use the following View Window parameters.

Xmin = −1	Ymin = −5
Xmax = 10	Ymax = 5
Xscale = 1	Yscale = 1

F5 (G-Solv)

`FOCS SYM DIR VTX LEN`
F1

F1 (FOCS)
(Calculates the focus.)

```
X=A(Y-K)²+H          ⊏
        ←

│
│
├────────────────────
│              FOCUS
X=3.25         Y=2
```

F5 (G-Solv)

`FOCS SYM DIR VTX LEN`
F4

F4 (VTX)
(Calculates the vertex.)

```
X=A(Y-K)²+H          ⊏
       ←

│
│
├────────────────────
│              VERTEX
X=3           Y=2
```
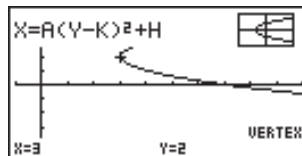
- When calculating two foci for an ellipse or hyperbolic graph, press ▶ to calculate the second focus. Pressing ◀ returns to the first focus.
- When calculating two vertexes for a hyperbolic graph, press ▶ to calculate the second vertex. Pressing ◀ returns to the first vertex.

● **To calculate the latus rectum**

Example **To determine the latus rectum for the parabola X = (Y − 2)² + 3**

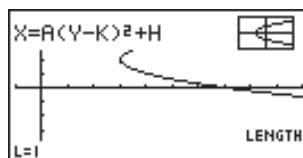Use the following View Window parameters.

Xmin = −1	Ymin = −5
Xmax = 10	Ymax = 5
Xscale = 1	Yscale = 1

F5 (G-Solv)

`FOCS SYM DIR VTX LEN`
F5

F5 (LEN)
(Calculates the latus rectum.)

X=A(Y-K)²+H

LENGTH

L=1

● **To calculate the center and radius**

Example   To determine the center and radius for the circle $X^2 + Y^2 - 2X - 2Y - 3 = 0$

Use the following View Window parameters.

Xmin = –6.3	**Ymin = –3.1**
Xmax = 6.3	**Ymax = 3.1**
Xscale = 1	**Yscale = 1**

F5 (G-Solv)

CNTR RADS

F1

F1 (CNTR)
(Calculates the center.)

AX²+AY²+BX+CY+D=0

+

CENTER

X=1   Y=1

F5 (G-Solv)

CNTR RADS

F2
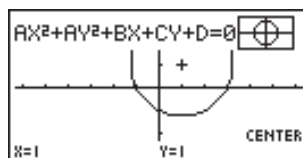
F2 (RADS)
(Calculates the radius.)

AX²+AY²+BX+CY+D=0

RADIUS

r=2.2360679775
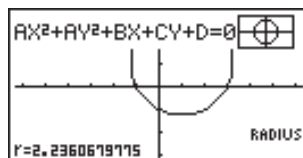
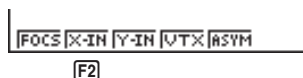● **To calculate the *x*- and *y*-intercepts**

___Example___   **To determine the *x*- and *y*-intercepts for the hyperbola**

$$\frac{(X - 1)^2}{2^2} - \frac{(Y - 1)^2}{2^2} = 1$$
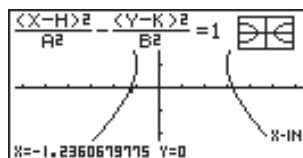
Use the following View Window parameters.

Xmin = –6.3	Ymin = –3.1
Xmax = 6.3	Ymax = 3.1
Xscale = 1	Yscale = 1

F5 (G-Solv)

FOCS X-IN Y-IN VTX ASYM
F2

F2 (X-IN)
(Calculates the *x*-intercept.)

$\frac{(X-H)^2}{A^2} - \frac{(Y-K)^2}{B^2} = 1$

X=-1.2360679775 Y=0

F5 (G-Solv)

FOCS X-IN Y-IN VTX ASYM
F3
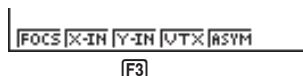
F3 (Y-IN)
(Calculates the *y*-intercept.)

$\frac{(X-H)^2}{A^2} - \frac{(Y-K)^2}{B^2} = 1$

Not Found
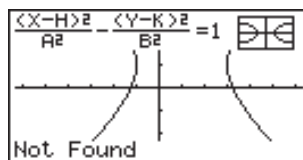
• Press ▶ to calculate the second set of *x*-/*y*-intercepts. Pressing ◀ returns to the first set of intercepts.

● **To draw and analyze the axis of symmetry and directrix**

___Example___   **To draw the axis of symmetry and directrix for the parabola**
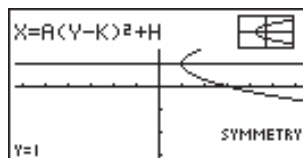**X = 2(Y – 1)² + 1**

Use the following View Window parameters.

Xmin = –6.3	Ymin = –3.1
Xmax = 6.3	Ymax = 3.1
Xscale = 1	Yscale = 1

F5 (G-Solv)

FOCS SYM DIR VTX LEN
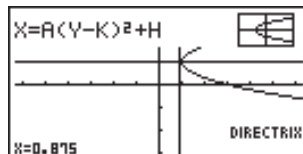F2

[F2] (SYM)

(Draws the axis of symmetry.)

[F5] (G-Solv)

[F3] (DIR)

(Draws the axis of directrix.)

● **To draw and analyze the asymptotes**

Example **To draw the asymptotes for the hyperbola**

$$\frac{(X - 1)^2}{2^2} - \frac{(Y - 1)^2}{2^2} = 1$$

Use the following View Window parameters.

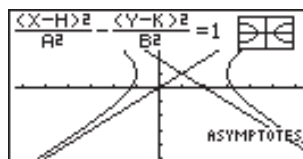|	
Xmin = –6.3	Ymin = –5
Xmax = 6.3	Ymax = 5
Xscale = 1	Yscale = 1

[F5] (G-Solv)

[F5] (ASYM)

(Draws the asymptotes.)

• Certain View Window parameters can produce errors in graph analysis result values.

• The message "**Not Found**" appears on the display when graph analysis is unable to produce a result.

• The following can result in inaccurate analysis results or may even make it impossible to obtain a solution at all.

• When the solution is tangent to the *x*-axis.

• When the solution is a point of tangency between two graphs.

232

14-4 Implicit Function Graphing Precautions

• Assigning the following types of values to variables contained in built-in function produces an error.

  (1) Parabola graph
      $A = 0$

  (2) Circle graph
      $R = 0$ for $(X - H)^2 + (Y - K)^2 = R^2$
      $A = 0$ for $AX^2 + AY^2 + BX + CY + D = 0$

  (3) Ellipse/hyperbola graph
      $A = 0$ or $B = 0$

• You cannot overwrite implicit function graphs.
• The calculator automatically clears the screen before drawing a new implicit function graph.
• You can use trace, scroll, zoom, or sketch after graphing an implicit function. However, an implicit function graph cannot be scrolled while using trace.
• You cannot incorporate graphing of an implicit function into a program.