4. Distribution (DIST)

There is a variety of different types of distribution, but the most well-known is "normal distribution," which is essential for performing statistical calculations. Normal distribution is a symmetrical distribution centered on the greatest occurrences of mean data (highest frequency), with the frequency decreasing as you move away from the center. Poisson distribution, geometric distribution, and various other distribution shapes are also used, depending on the data type.

Certain trends can be determined once the distribution shape is determined. You can calculate the probability of data taken from a distribution being less than a specific value.

For example, distribution can be used to calculate the yield rate when manufacturing some product. Once a value is established as the criteria, you can calculate normal probability when estimating what percent of the products meet the criteria. Conversely, a success rate target (80% for example) is set up as the hypothesis, and normal distribution is used to estimate the proportion of the products will reach this value.

Normal probability density calculates the probability density of normal distribution that data taken from a specified x value.

Normal distribution probability calculates the probability of normal distribution data falling between two specific values.

Inverse cumulative normal distribution calculates a value that represents the location within a normal distribution for a specific cumulative probability.

Student-t probability density calculates the probability density of t distribution that data taken from a specified x value.

Student-t distribution probability calculates the probability of t distribution data falling between two specific values.

Like t distribution, distribution probability can also be calculated for χ^2, F, **Binomial**, **Poisson**, and **Geometric** distributions.

On the initial STAT2 Mode screen, press [F5] (DIST) to display the distribution menu, which contains the following items.

- [F5] (DIST) [1] (Norm) ... Normal distribution (p.44)

  [2] (T) ... Student-t distribution (p.48)

  [3] (χ^2) ... χ^2 distribution (p.50)

  [4] (F) ... F distribution (p.53)

  [5] (Binmal) ... Binomial distribution (p.57)

  [6] (Poissn) ... Poisson distribution (p.60)

  [7] (Geo) ... Geometric distribution (p.62)

●**Common Distribution Functions**

After drawing a graph, you can use the P-CAL function to calculate an estimated p-value for a particular x value.

The following is the general procedure for using the P-CAL function.

1. After drawing a graph, press [F1] (P-CAL) to display the x value input dialog box.

2. Input the value you want for x and then press [EXE].

• This causes the x and p values to appear at the bottom of the display, and moves the pointer to the corresponding point on the graph.

3. Pressing [X,θ,T] or a number key at this time causes the x value input dialog box to reappear so you can perform another estimated value calculation if you want.

4. After you are finished, press [ESC] to clear the coordinate values and the pointer from the display.

Executing an analysis function automatically stores the x and p values in alpha variables X and P, respectively.

■ **Normal Distribution**

●**Normal Probability Density**

Normal probability density calculates the probability density of nomal distribution that data taken from a specified x value. Normal probability density is applied to standard normal distribution.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (\sigma > 0)$$

Perform the following key operation from the statistical data list.

F5 (DIST)
1 (Norm)
1 (P.D)

```
Normal P.D
x        :0
σ        :1
μ        :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

x .... data

σ .... population standard deviation ($\sigma > 0$)

μ .... population mean

Save Res .... list for storage of calculation results (None or List 1 to 20)

Execute .... executes a calculation or draws a graph

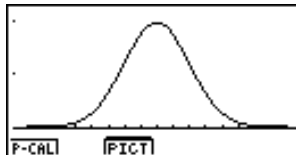• Specifying $\sigma = 1$ and $\mu = 0$ specifies standard normal distribution.

After setting all the parameters, align the cursor with [Execute] and then press one of the function keys shown below to perform the calculation or draw the graph.

• F1 (CALC) ... Performs the calculation.

• F6 (DRAW) ... Draws the graph.

Calculation Result Output Example

```
Normal P.D
    p=0.17603266
```

```
P-CAL   PICT
```

• p ... normal probability density

V-Window settings for graph drawing are set automatically when the SET UP screen's [Stat Wind] setting is [Auto]. Current V-

Window settings are used for graph drawing when the [Stat Wind] setting is [Manual].

●Normal Distribution Probability

Normal distribution probability calculates the probability of normal distribution data falling between two specific values.

$$p = \frac{1}{\sqrt{2\pi}\sigma} \int_{a}^{b} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

a : lower boundary
b : upper boundary

Perform the following key operation from the statistical data list.

F5   (DIST)
1   (Norm)
2   (C.D)

```
Normal C.D
Lower    :0
Upper    :0
σ        :1
μ        :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

Lower .... lower boundary

Upper .... upper boundary

σ .... population standard deviation ($\sigma > 0$)

μ .... population mean

Save Res .... list for storage of calculation results (None or List 1 to 20)

Execute .... executes a calculation

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• F1 (CALC) ... Performs the calculation.

There is no graphing for normal distribution probability.

Calculation Result Output Example

```
Normal C.D
p    =0.09184805
z:Low=1
z:Up =1.5
```

p .... normal distribution probability

z:Low .... z:Low value (converted to standardize z score for lower value)

z:Up .... z:Up value (converted to standardize z score for upper value)

•Inverse Cumulative Normal Distribution

Inverse cumulative normal distribution calculates a value that represents the location within a normal distribution for a specific cumulative probability.

Tail : Left $\int_{-\infty}^{\alpha} f(x)dx = p$
upper
boundary of
integration
interval
$\alpha = ?$

Tail : Right $\int_{\alpha}^{+\infty} f(x)dx = p$
lower
boundary of
integration
interval
$\alpha = ?$

Tail : Central $\int_{\alpha}^{\beta} f(x)dx = p$
upper and
lower
boundaries
of integration
interval
$\alpha = ?$ $\beta = ?$

Specify the probability and use this formula to obtain the integration interval.

Perform the following key operation from the statistical data list.

F5 (DIST)
1 (Norm)
3 (Invrse)

```
Inverse Normal
Tail    :Left
Area    :0
σ       :1
μ       :0
Save Res:None
Execute
LEFT RIGHT CENTR
```

Data is specified using parameter specification. The following shows the meaning of each item.

Tail .... probability value tail specification (Left, Right, Central)

Area .... probability value ($0 \leq$ Area ≤ 1)

σ .... population standard deviation ($\sigma > 0$)

μ .... population mean

Save Res .... list for storage of calculation results (None or List 1 to 20)

Execute .... executes a calculation

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

  • [F1] (CALC) ... Performs the calculation.

Calculation Result Output Examples

```
Inverse Normal
    x=1.64485363
```

```
Inverse Normal
x:Low=-0.0627067
x:Up =0.06270677
```

x .... inverse cumulative normal distribution
(Tail:Left upper boundary of integration interval)
(Tail:Right lower boundary of integration interval)
(Tail:Central upper and lower boundaries of integration interval)

There is no graphing for inverse cumulative normal distribution.

■ **Student-*t* Distribution**

●**Student-*t* Probability Density**

Student-*t* probability density calculates the probability density of *t* distribution that data taken from a specified x value.

$$f(x) = \frac{\Gamma\left(\dfrac{df+1}{2}\right)}{\Gamma\left(\dfrac{df}{2}\right)} \frac{\left(1+\dfrac{x^2}{df}\right)^{-\frac{df+1}{2}}}{\sqrt{\pi df}}$$

Perform the following key operation from the statistical data list.

[F5] (DIST)
[2] (T)
[1] (P.D)

```
Student-t P.D
x       :0
df      :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

x .... data

df .... degrees of freedom (*df* >0)

Save Res .... list for storage of calculation results (None or List 1 to 20)

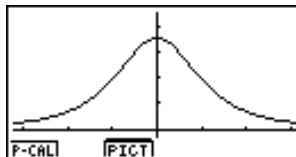Execute .... executes a calculation or draws a graph

After setting all the parameters, align the cursor with [Execute] and then press one of the function keys shown below to perform the calculation or draw the graph.

• [F1] (CALC) ... Performs the calculation.

• [F6] (DRAW) ... Draws the graph.

Calculation Result Output Example

```
Student-t P.D
    P=0.19245009
```


p ... Student-*t* probability density

Current V-Window settings are used for graph drawing when the SET UP screen's [Stat Wind] setting is [Manual]. The V-Window settings below are set automatically

when the [Stat Wind] setting is [Auto].

Xmin = –3.2, *Xmax* = 3.2, *Xscale* = 1,

Ymin = –0.1, *Ymax* = 0.45, *Yscale* =0.1

●**Student-*t* Distribution Probability**

Student-*t* distribution probability calculates the probability of *t* distribution data falling between two specific values.

$$p = \frac{\Gamma\left(\dfrac{df+1}{2}\right)}{\Gamma\left(\dfrac{df}{2}\right)\sqrt{\pi df}} \int_a^b \left(1+\frac{x^2}{df}\right)^{-\frac{df+1}{2}} dx$$

a : lower boundary
b : upper boundary

Perform the following key operation from the statistical data list.

[F5] (DIST)
[2] (T)
[2] (C.D)

```
Student-t C.D
Lower   :0
Upper   :0
df      :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

Lower .... lower boundary
Upper .... upper boundary
df .... degrees of freedom ($df > 0$)
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• [F1] (CALC) ... Performs the calculation.

There is no graphing for Student-*t* distribution probability.

Calculation Result Output Example

```
Student-t C.D
P    =0.0548831
t:Low=1.7
t:Up =1ε+99
```

p ... Student-t distribution probability

t:Low ... t:Low value (input lower value)

t:Up ... t:Up value (input upper value)

■ χ^2 Distribution

●χ^2 Probability Density

χ^2 probability density calculates the probabilitty density function for the χ^2 distribution at a specified x value.

$$f(x) = \frac{1}{\Gamma\left(\frac{df}{2}\right)}\left(\frac{1}{2}\right)^{\frac{df}{2}} x^{\frac{df}{2}-1} e^{-\frac{x}{2}}$$

Perform the following key operation from the statistical data list.

[F5](DIST)
[3](χ^2)
[1](P.D)

```
χ² P.D
x        :0
df       :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

x .... data

df .... degrees of freedom (positive integer)

Save Res .... list for storage of calculation results (None or List 1 to 20)

Execute .... executes a calculation or draws a graph

After setting all the parameters, align the cursor with [Execute] and then press one of the function keys shown below to perform the calculation or draw the graph.

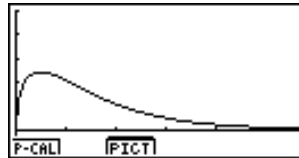• [F1](CALC) ... Performs the calculation.
• [F6](DRAW) ... Draws the graph.

Calculation Result Output Example

p ... χ^2 probability density

Current V-Window settings are used for graph drawing when the SET UP screen's [Stat Wind] setting is [Manual]. The V-Window settings below are set automatically

when the [Stat Wind] setting is [Auto].

$Xmin = 0$, $Xmax = 11.5$, $Xscale = 2$, $Ymin = -0.1$, $Ymax = 0.5$, $Yscale = 0.1$

•χ² Distribution Probability

χ² distribution probability calculates the probability of χ² distribution data falling between two specific values.

$$p = \frac{1}{\Gamma\left(\frac{df}{2}\right)}\left(\frac{1}{2}\right)^{\frac{df}{2}} \int_a^b x^{\frac{df}{2}-1} e^{-\frac{x}{2}}\, dx$$

a : lower boundary
b : upper boundary

Perform the following key operation from the statistical data list.

$\boxed{F5}$(DIST)
$\boxed{3}$(χ²)
$\boxed{2}$(C.D)

```
x² C.D
Lower    :0
Upper    :0
df       :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

Lower .... lower boundary
Upper .... upper boundary
df .... degrees of freedom (positive integer)
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• $\boxed{F1}$(CALC) ... Performs the calculation.

There is no graphing for χ² distribution probability.

Calculation Result Output Example

$$x^2 \quad \text{C.D} \atop p=0.22884525$$

p ... χ^2 distribution probability

■ *F* **Distribution**

• *F* **Probability Density**

F probability density calculates the probability density function for the *F* distribution at a specified x value.

$$f(x) = \frac{\Gamma\left(\dfrac{n+d}{2}\right)}{\Gamma\left(\dfrac{n}{2}\right)\Gamma\left(\dfrac{d}{2}\right)} \left(\dfrac{n}{d}\right)^{\frac{n}{2}} x^{\frac{n}{2}-1} \left(1 + \dfrac{nx}{d}\right)^{-\frac{n+d}{2}}$$

Perform the following key operation from the statistical data list.

[F5](DIST)
[4](F)
[1](P.D)

```
F P.D
x      :0
n:df   :0
d:df   :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

x .... data

n:*df* .... numerator degrees of freedom (positive integer)

d:*df* .... denominator degrees of freedom (positive integer)

Save Res .... list for storage of calculation results (None or List 1 to 20)

Execute .... executes a calculation or draws a graph

After setting all the parameters, align the cursor with [Execute] and then press one of the function keys shown below to perform the calculation or draw the graph.
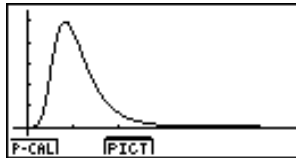
• [F1](CALC) ... Performs the calculation.

• [F6](DRAW) ... Draws the graph.

Calculation Result Output Example

p ... *F* probability density

V-Window settings for graph drawing are set automatically when the SET UP screen's [Stat Wind] setting is [Auto]. Current V-Window settings are used for graph drawing when the [Stat Wind] setting is [Manual].

• *F* **Distribution Probability**

F distribution probability calculates the probability of *F* distribution data falling between two specific values.

$$p = \frac{\Gamma\left(\dfrac{n+d}{2}\right)}{\Gamma\left(\dfrac{n}{2}\right)\Gamma\left(\dfrac{d}{2}\right)} \left(\frac{n}{d}\right)^{\frac{n}{2}} \int_a^b x^{\frac{n}{2}-1}\left(1+\frac{nx}{d}\right)^{-\frac{n+d}{2}} dx$$

a : lower boundary
b : upper boundary

Perform the following key operation from the statistical data list.

F5 (DIST)
4 (F)
2 (C.D)

```
F C.D
Lower    :0
Upper    :0
n:df     :0
d:df     :0
Save Res:None
Execute
```

Data is specified using parameter specification. The following shows the meaning of each item.

Lower .... lower boundary

Upper .... upper boundary

n:*df* .... numerator degrees of freedom (positive integer)

d:*df* .... denominator degrees of freedom (positive integer)

Save Res .... list for storage of calculation results (None or List 1 to 20)

Execute .... executes a calculation

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• F1 (CALC) ... Performs the calculation.

There is no graphing for *F* distribution probability.

Calculation Result Output Example

```
F C.D
   P=0.91400535
```

p ... F distribution probability

■ Binomial Distribution

● Binomial Probability

Binomial probability calculates a probability at specified value for the discrete binomial distribution with the specified numtrials and probability of success on each trial.

$$f(x) = {}_nC_x p^x (1-p)^{n-x} \qquad (x = 0, 1, \cdots\cdots, n) \quad p : \text{success probability}$$
$$(0 \leq p \leq 1)$$
$$n : \text{number of trials}$$

Perform the following key operation from the statistical data list.

F5 (DIST)
5 (Binmal)
1 (P.D)

```
Binomial P.D
Data    :List
List    :List1
Numtrial:0
P       :0
Save Res:None
Execute
LIST VAR
```

The following shows the meaning of each item when data is specified using list specification.

Data .... data type
List .... list whose contents you want to use as specified data
Numtrial .... number of trials
p .... success probability $(0 \leq p \leq 1)$
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

The following shows the meaning of parameter data specification items that are different from list data specification.

| x | :0 | |

x .... integer from 0 to n

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• F1 (CALC) ... Performs the calculation.

There is no graphing for binomial distribution.

Calculation Result Output Example

```
Binomial P.D
1 0.3599
2 0.4799
3 0.1599

              0.36
```

```
Binomial P.D
  P=0.48
```

p ... Binomial probability

●Binomial Cumulative Density

Binomial cumulative density calculates a cumulative probability at specified value for the discrete binomial distribution with the specified numtrials and probability of success on each trial.

Perform the following key operation from the statistical data list.

[F5] (DIST)

[5] (Binmal)

[2] (C.D)

```
Binomial C.D
Data    :List
List    :List1
Numtrial:0
P       :0
Save Res:None
Execute
LIST VAR
```

The following shows the meaning of each item when data is specified using list specification.

Data .... data type

List .... list whose contents you want to use as specified data

Numtrial .... number of trials

p .... success probability $(0 \leqq p \leqq 1)$

Save Res .... list for storage of calculation results (None or List 1 to 20)

Execute .... executes a calculation

The following shows the meaning of parameter data specification item that is different from list data specification.

|x   :0 |

x .... integer from 0 to n

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• F1(CALC) ... Performs the calculation.

Calculation Result Output Example

```
Binomial C.D
 1 |      |
 2| 0.999
 3| 0.9843
 4| 0.8964
 5|0.6328
                    1
```

```
Binomial C.D
   p=0.984375
```

p ... Binomial cumulative density

■ Poisson Distribution

●Poisson Probability

Poisson probability calculates a probability at specified value for the discrete Poisson distribution with the specified mean.

$$f(x) = \frac{e^{-\mu}\mu^x}{x!} \qquad (x = 0, 1, 2, \cdots) \qquad \mu : \text{population mean } (\mu > 0)$$

Perform the following key operation from the statistical data list.

F5 (DIST)
6 (Poissn)
1 (P.D)

```
Poisson P.D
Data    :List
List    :List1
µ       :0
Save Res:None
Execute

LIST VAR
```

The following shows the meaning of each item when data is specified using list specification.

Data .... data type
List .... list whose contents you want to use as specified data
μ .... population mean ($\mu > 0$)
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

The following shows the meaning of parameter data specification item that is different from list data specification.

```
|x        :0                    |
```

x .... ($x \geqq 0$)

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• F1 (CALC) ... Performs the calculation.

Calculation Result Output Example

```
Poisson P.D
1 0.2231
2 0.3346
3 0.251

        0.2231301601
```

```
Poisson P.D
   p=0.25102143
```

p ... Poisson probability

There is no graphing for Poisson distribution.

● Poisson Cumulative Density

Poisson cumulative density calculates a cumulative probability at specified value for the discrete Poisson distribution with the specified mean.

Perform the following key operation from the statistical data list.

> [F5] (DIST)
> [6] (Poissn)
> [2] (C.D)

```
Poisson C.D
Data    :List
List    :List1
μ       :0
Save Res:None
Execute

LIST|VAR
```

The following shows the meaning of each item when data is specified using list specification.

Data .... data type
List .... list whose contents you want to use as specified data
μ .... population mean ($\mu > 0$)
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a caluculation

The following shows the meaning of parameter data specification item that is different from list data specification.

|x   :0 |

x .... ($x \geqq 0$)

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• [F1] (CALC) ... Performs the calculation.

Calculation Result Output Example

```
Poisson C.D
1 5.1E-6
2 7.9E-5
3 5.2E-4
4 2.2E-3
5 7.6E-3
        6.144212353E-06
```

```
Poisson C.D
   p=0.15502778
```

p ... Poisson cumulative density

■ Geometric Distribution

●Geometric Probability

Geometric probability calculates a probability at specified value, the number of the trial on which the first success occurs, for the discrete geometric distribution with the specified probability of success.

$$f(x) = p(1-p)^{x-1} \qquad (x = 1, 2, 3, \cdots)$$

Perform the following key operation from the statistical data list.

[F5](DIST)
[7](Geo)
[1](P.D)

```
Geometric P.D
Data    :List
List    :List1
p       :0
Save Res:None
Execute

LIST VAR
```

The following shows the meaning of each item when data is specified using list specification.

Data .... data type
List .... list whose contents you want to use as specified data
p .... success probability $(0 \leq p \leq 1)$
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

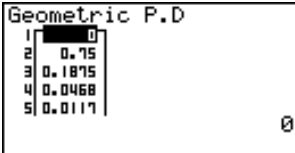The following shows the meaning of parameter data specification item that is different from list data specification.

|x   :0 |

x .... positive integer $(x \geq 1)$

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.
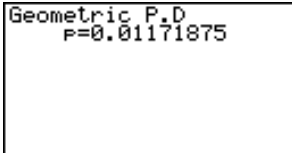
• [F1](CALC) ... Performs the calculation.

Calculation Result Output Example

```
Geometric P.D
1      0
2  0.75
3 0.1875
4 0.0468
5 0.0117
                 0
```
```
Geometric P.D
p=0.01171875
```

p ... Geometric probability

There is no graphing for geometric distribution.

Positive integer number is calculated whether list data (Data:List) or x value (Data:variable) is specified.

●**Geometric Cumulative Density**

Geometric cumulative density calculates a cumulative probability at specified value, the number of the trial on which the first success occurs, for the discrete geometric distribution with the specified probability of success.

Perform the following key operation from the statistical data list.

[F5](DIST)
[7](Geo)
[2](C.D)

```
Geometric C.D
Data     :List
List     :List1
P        :0
Save Res:None
Execute

LIST VAR
```

The following shows the meaning of each item when data is specified using list specification.

Data .... data type
List .... list whose contents you want to use as specified data
p .... success probability ($0 \leqq p \leqq 1$)
Save Res .... list for storage of calculation results (None or List 1 to 20)
Execute .... executes a calculation

The following shows the meaning of parameter data specification item that is different from list data specification.

|x   :0 |

x .... positive integer ($x \geqq 1$)

After setting all the parameters, align the cursor with [Execute] and then press the function key shown below to perform the calculation.

• [F1](CALC) ... Performs the calculation.

Calculation Result Output Example

```
Geometric C.D
1  0.75
2 0.8749
3 0.9375

           0.75
```

```
Geometric C.D
P=0.875
```

p ... Geometric cumulative density

Positive integer number is calculated whether
list data (Data:List) or x value (Data:variable)
is specified.