Recursion Table and Graph

You can input two formulas for any of the three following types of recursion, which you can then use to generate a table and draw graphs.

• General term of sequence \( \{a_n\} \), made up of \( a_n \) and \( n \)

• Formulas for linear recursion between two terms, made up of \( a_{n+1}, a_n, \) and \( n \)

• Formulas for linear recursion between three terms, made up of \( a_{n+2}, a_{n+1}, a_n, \) and \( n \)

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16-1 Before Using the Recursion Table and Graph Function

● To enter the RECUR Mode

On the Main Menu, select the **RECUR** icon and enter the RECUR Mode. This causes the Recursion Menu to appear.

- All recursion formulas that are stored in memory appear in the Recursion Menu.
- {SEL+C} ... {menus for control of table generation and graph color}
- {SEL} ... {recursion formula generation/non-generation status}
- {DEL} ... {recursion formula delete}
- {TYPE} ... {recursion formula type specification}
- {n, an} ... {menu for input of variable n and general terms an and bn}
- {RANG} ... {screen for setting of table range}
- {TABL} ... {recursion formula table generation}

● To specify the recursion formula type

Before inputting a recursion formula, you must first specify its type.

1. In the Recursion Menu, press **F3** (TYPE).

   - In this display, “an = An + B” is the general term (an = A \times n + B) of {an}.

2. Press the function key for the recursion formula type you want to set.
   - {an}/(an+1)/(an+2) ... {general term of sequence {an}}/(linear recursion between two terms)/(linear recursion between three terms)
16-2 Inputting a Recursion Formula and Generating a Table

Example 1 To input $a_{n+1} = 2a_n + 1$ and generate a table of values as the value of $n$ change from 1 to 6

Make $a_1 = 1$.

1. Specify the recursion formula type as linear recursion between two terms and then input the formula.

2. Press $\text{w5} (\text{RANG})$ to display the table range setting screen, which contains the following items.

- $\{a_0\}/\{a_1\}$ ... setting of value for $\{a_0(b_0)/\{a_1(b_1)\}$

The table range settings specify the conditions that control the value of variable $n$ in the recursion formula, and the initial term of the numeric value table. You should also specify a starting point for the pointer when drawing a convergence/divergence graph (WEB graph) for a formula for linear recursion between two terms.

Start ...................... Starting value of variable $n$
End ...................... Ending value of variable $n$

- $a_0, b_0$ ............... Value of 0th term $a_0/b_0$ ($a_1, b_1$ .... Value of 1st term $a_1/b_1$)
- $a_n, b_n$Str ........ Pointer starting point for convergence/divergence graph (WEB graph)

- The value of variable $n$ increments by 1.

3. Specify the range of the table.

4. Display the table of the recursion formula. At this time, a menu of table functions appears at the bottom of the screen.
Displayed cell values show positive integers up to six digits, and negative integers up to five digits (one digit used for negative sign). Exponential display can use up to three significant digits.

You can see the entire value assigned to a cell by using the cursor keys to move the highlighting to the cell whose value you want to view.

You can also display the sums of the terms ($\Sigma a_n$ or $\Sigma b_n$) by turning on $\Sigma$ Display.

Example 2 To input $a_{n+2} = a_{n+1} + a_n$ (Fibonacci series) and generate a table of values as the value of $n$ change from 1 to 6

Make $a_1 = 1$ and $a_2 = 1$.

1. Specify the recursion formula type as linear recursion between three terms and then input the formula.

   $F3$ (TYPE) $F3$ ($a_{n+2}$) $F4$ ($n, a_{n..}$) $\text{Recursion}$

   $a_{n+2} = a_{n+1} + a_n$

2. Press $w_5$ and then press $F3$ (RANG) to display the table range setting screen, which contains the following items.

   * $\{a_0\}/\{a_1\}$ ... setting of value for $\{a_0 \ (b_0) \text{ and } a_1 \ (b_1)\}/\{a_1 \ (b_1) \text{ and } a_2 \ (b_2)\}$

   The table range settings specify the conditions that control the value of variable $n$ in the recursion formula, and the initial term of the numeric value table.

   Start .................. Starting value of variable $n$

   End .................... Ending value of variable $n$

   $a_0, a_1, a_2$ .......... Values of 0th term $a_0/b_0$, 1st term $a_1/b_1$, and 2nd term $a_2/b_2$.

   The value of variable $n$ increments by 1.

3. Specify the range of the table.

   $F2$ ($a_1$)
   
   $1 \ 6 \ 5 \ 1 \ 1 \ 1$
4. Display the table of the recursion formula. At this time, a menu of table functions appears at the bottom of the screen.

- There can be only one recursion table stored in memory at one time.
- Except for linear expression \( n \), any of the following can be input for general term \( \{a_n\} \) to generate a table: exponential expressions (such as \( a_n = 2^n - 1 \)), fractional expressions (such as \( a_n = (n + 1)/n \)), irrational expressions (such as \( a_n = \sqrt{n} - \sqrt{n - 1} \)), trigonometric expressions (such as \( a_n = \sin 2\pi n \)).

- Note the following points when specifying a table range.
  - If a negative value is specified as a start or end value, the calculator drops the negative sign. If a decimal value or fraction is specified, the unit uses only the integer part of the value.
  - When Start = 0 and \( a_1/b_1 \) is selected as the initial term, the calculator changes to Start = 1 and generates the table.
  - When Start > End, the calculator swaps the Start and End values and generates the table.
  - When Start = End, the calculator generates a table for Start values only.
  - If the start value is very large, it may take a long time to generate a table for linear recursion between two terms and linear recursion between three terms.
  - Changing the angle unit setting while a table generated from a trigonometric expression is on the display does not cause the displayed values to change. To cause the values in the table to be updated using the new setting, display the table, press [F1] (FORM), change the angle unit setting, and then press [F6] (TABL).
To specify the generation/non-generation status of a formula

Example: To specify generation of a table for recursion formula $a_{n+1} = 2a_n + 1$ while there are two formulas stored.

- Each press of F1 (SEL) toggles a table between generation and non-generation.

To change the contents of a recursion formula

Changing the contents of a recursion formula causes the values in the table to be updated using the current table range settings.

Example: To change $a_{n+1} = 2a_n + 1$ to $a_{n+1} = 2a_n - 3$

- To delete a recursion formula

1. Use ▲ and ◀ to highlight the formula you want to delete. Press F2 (DEL).
2. Press F1 (YES) to delete the formula or F6 (NO) to abort the operation without deleting anything.
You get a choice of four options for editing tables and drawing graphs.

- Deletion of a recursion formula table
- Drawing of a connect type graph
- Drawing of a plot type graph
- Drawing of a graph and analysis of convergence/divergence (WEB)

You can access these options from the function menu that appears at the bottom of the screen whenever a table is displayed.

- \{FORM\} ... \{returns to Recursion Menu\}
- \{DEL\} ... \{table delete\}
- \{WEB\} ... \{convergence/divergence (WEB) graph draw\}
- \{G-CON\}/\{G-PLT\} ... \{connected type\}/\{draw plot type\} recursion graph draw

The \{WEB\} item is available only when a table generated using a formula for linear recursion between two terms ($a_{n+1} =, b_{n+1} =$) is on the display.

**To delete a recursion table**

1. Display the recursion table you want to delete and then press \([F2]\) (DEL).

2. Press \([F1]\) (YES) to delete the table or \([F6]\) (NO) to abort the operation without deleting anything.

**Before Drawing a Graph for a Recursion Formula**

You must first specify the following.

- Graph color (blue, orange, green) \{BLUE\}/\{ORNG\}/\{GRN\}
- Draw/non-draw status of for the recursion formula \{SEL\}
- The type of data to be plotted \Sigma Display
To specify the color of the graph ($\text{BLUE}/\text{ORNG}/\text{GRN}$)

The default color for a graph is blue. Use the following procedure to change the graph color to orange or green.

1. Display the Recursion Menu and then use $\uparrow$ and $\downarrow$ to highlight the formula whose graph color you want to change.

2. Press F1 (SEL+C).

3. Press the function key that corresponds to the color you want to specify.

To specify the draw/non-draw status of a formula ($\{\text{SEL}\}$)

There are two options for the draw/non-draw status of a recursion formula graph.

- Draw the graph for the selected recursion formula only
- Overlay the graphs for both recursion formulas

To specify the draw/non-draw status, use the same procedure as that for specifying generation/non-generation status.

To specify the type of data to be plotted ($\Sigma$ Display: On)

You can specify one of two types of data for plotting.

- $a_n$ on the vertical axis, $n$ on the horizontal axis
- $\Sigma a_n$ on the vertical axis, $n$ on the horizontal axis

In the function menu that appears while a table is on the display, press F5 (G·CON) or F6 (G·PLT) to display the plot data menu.

- $\{a_n\}/\{\Sigma a_n\}$ ... $\{a_n\}/\{\Sigma a_n\}$ on vertical axis, $n$ on horizontal axis

**Example 1** Draw a graph of $a_{n+1} = 2a_n + 1$ with $a_n$ on the vertical axis and $n$ on the horizontal axis, and with the points connected.

Set the following parameters in the View Window.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xmin</td>
<td>0</td>
</tr>
<tr>
<td>Ymin</td>
<td>0</td>
</tr>
<tr>
<td>Xmax</td>
<td>6</td>
</tr>
<tr>
<td>Ymax</td>
<td>65</td>
</tr>
<tr>
<td>Xscale</td>
<td>1</td>
</tr>
<tr>
<td>Yscale</td>
<td>5</td>
</tr>
</tbody>
</table>

F6 (TABL) F5 (G·CON)

(Selects connected type.)

F1 ($a_n$)

(Draws graph with $a_n$ on the vertical axis.)
Example 2  Draw a graph of \( a_{n+1} = 2a_n + 1 \) with \( \Sigma a_n \) on the vertical axis and \( n \) on the horizontal axis, and with the points unconnected.

Use the same View Window parameters as those provided in Example 1.

\[ \text{F6 (TABL) F6 (G·PLT)} \]
(Selects plot type.)

\[ \text{F6 (} \Sigma \text{a)} \]
(Draws graph with \( \Sigma a_n \) on the vertical axis.)

- To input a different recursion formula after a graph is drawn, press \( \text{SHIFT} \) \( \text{QUIT} \). This displays the Recursion Menu where you can input a new formula.

### Drawing a Convergence/Divergence Graph (WEB graph)

With this feature, you can draw a graph of \( a_{n+1} = f(a_n) \) where \( a_{n+1} \) and \( a_n \) are the terms of linear recursion between two terms, substituted respectively for \( y \) and \( x \) in the function \( y = f(x) \). The resulting graph can then be viewed to determine whether or not the graph is convergent or divergent.

**Example 1**  To determine whether or not the recursion formula \( a_{n+1} = -3a_n^2 + 3a_n \) is convergent or divergent.

Use the following table range.

- \( a_0 = 0.01 \) \( a_n \) Str = 0.01
- \( b_0 = 0.11 \) \( b_n \) Str = 0.11

Use the following View Window parameters.

- \( X_{\text{min}} = 0 \) \( Y_{\text{min}} = 0 \)
- \( X_{\text{max}} = 1 \) \( Y_{\text{max}} = 1 \)
- \( X_{\text{scale}} = 1 \) \( Y_{\text{scale}} = 1 \)

This example assumes that the following two recursion formulas are already stored in memory.

1. Press \( \text{F6 (TABL) F4 (WEB)} \) to draw the graph.
2. Press \( \text{Ex} \), and the pointer appears at the pointer start point \((a, \text{Str} = 0.01)\).

\[
\begin{align*}
\text{an+1} &= -3a_n^2 + 3a_n \\
\text{X} &= 0.01 \\
\text{Y} &= 0
\end{align*}
\]

• The Y value for the pointer start point is always 0.

3. Each press of \( \text{Ex} \) draws web-like lines on the display.

\[
\begin{align*}
\text{End} \\
\text{Ex} \downarrow \\
\text{Ex} \\
\text{End} \downarrow \\
\text{Ex}
\end{align*}
\]

This graph indicates that recursion formula \( a_{n+1} = -3a_n^2 + 3a_n \) is convergent.

**Example 2**  
To determine whether or not the recursion formula \( b_{n+1} = 3b_n + 0.2 \) is convergent or divergent.

Use the following table range.

- **Start** = 0  
- **End** = 6  
- \( b_0 = 0.02 \)  
- \( b_n \text{ Str} = 0.02 \)

Use the View Window parameters from Example 1.

1. Press \( \text{F6 (TABL)} \text{ F4 (WEB)} \) to draw the graph.
2. Press \( w \) and then either \( \uparrow \) or \( \downarrow \) to make the pointer appear at the pointer start point \((b_n, \text{Str} = 0.02)\).

\[ b_{n+1} = 3b_n + 0.2 \]

- The Y value for the pointer start point is always 0.

3. Each press of \( \text{EX} \) draws web-like lines on the display.

This graph indicates that recursion formula \( b_{n+1} = 3b_n + 0.2 \) is divergent.

- Inputting \( b_n \) or \( n \) for the expression \( a_{n+1} \), or Inputting \( a_n \) or \( n \) for the expression \( b_{n+1} \) for linear recursion between two terms causes an error.
Drawing a Recursion Formula Graph Using Dual Screen

Selecting “T+G” for the Dual Screen item of the setup screen makes it possible to display both the graph and its numerical table of values.

**Example** To draw the graph of \( a_{n+1} = 2a_n + 1 \) from Example 1, displaying both the graph and its table

Display the setup menu and specify “T+G” for Dual Screen. Press EXIT.

\( F_6 \) (TABL)
(Shows the table.)

\( F_6 \) (G-PLT)
(Draws plot type graph.)

Pressing \( \text{SHIFT} \ F_6 \) \((G↔T)\) causes the graph on the left side of the Dual Screen to fill the entire display. Note that you cannot use the sketch function while a graph is displayed using \( \text{SHIFT} \ F_6 \) \((G↔T)\).