Differential Equations (DIFF EQ) Software for the ALGEBRA FX 2.0

- 1. Using the DIFF EQ Mode
- 2. Differential Equations of the First Order
- 3. Linear Differential Equations of the Second Order
- 4. Differential Equations of the Nth Order
- 5. System of First Order Differential Equations

1. Using the DIFF EQ Mode

You can solve differential equations numerically and graph the solutions. The general procedure for solving a differential equation is described below.

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

Differential Equation
F1:1st Order Equation
F2:2nd Order Lin Eq
F3:Nth Order Equation
F4:1st Order System
F5:Recall Last
Ist Tand N-thlsysTRCL

Execution

- 2. Select the differential equation type.
 - F1 (1st) Four types of first order differential equations
 - F2 (2nd) Second order linear differential equations
 - [F3] (N-th) Differential equations of the first order through ninth order
 - F4 (SYS) System of the first order differential equations
 - F5 (RCL) Displays a screen for recalling a previous differential equation.
 - With F1 (1st), you need to make further selections of differential equation type. See "Differential equations of the first order" for more information.
 - With F3 (N-th), you also need to specify the order of the differential equation, from 1 to 9.
 - With [F4] (SYS), you also need to specify the number of unknowns, from 1 to 9.
- 3. Enter the differential equation.
- 4. Specify the initial values.
- 5. Press F5 (SET) and select 1 (Param) to display the Parameter screen. Specify the calculation range. Make the parameter settings you want.
 - h...... Step size for the classical Runge-Kutta method (fourth order)
 - Step Number of steps for graphing*1 and storing data in LIST.
 - SF The number of slope field columns displayed on the screen (0 100).
 The slope fields can be displayed only for differential equations of the first order.



*1When graphed for the first time, a function is always graphed with every step. When the function is graphed again, however, it is graphed according to a value of Step. For example, when Step is set to 2, the function is graphed with every two steps.



6. Specify variables to graph or to store in LIST.

Press [F5] (SET) and select 2 (Output) to display the list setting screen.

 $x, y, y^{(1)}, y^{(2)},, y^{(8)}$ stand for the independent variable, the dependent variable, the first order derivative, the second order derivative,, and the eighth order derivative, respectively.

1st, 2nd, 3rd,, 9th stand for the initial values in order.

To specify a variable to graph, select it using the cursor keys (\triangle, \bigcirc) and press (E)

To specify a variable to store in LIST, select it using the cursor keys ($\textcircled{\bullet}$, $\textcircled{\bullet}$) and press [F2] (LIST).

7. Press (PT) (V-Window) to display the V-Window setting screen. Before you solve a differential equation, you need to make V-Window settings.

Xmin ... x-axis minimum value

max ... x-axis maximum value

scale ... x-axis value spacing

dot ... value corresponding to one x-axis dot

Ymin ... y-axis minimum value

 $\max \dots y$ -axis $\max i mum value$

scale ... y-axis value spacing

- 8. Press F6 (CALC) to solve the differential equation.
 - The calculated result is graphed or stored in the list.







- # Only the slope fields are displayed if you do not input initial values or if you input the wrong type of initial values.
- # An error occurs if you set SF to zero and you do not input the initial values, or if you input the initial values inappropriately.
- # You are advised to input parentheses and a multiplication sign between a value and an expression in order to prevent calculation errors
- # Do not confuse the key and the key.

 A syntax error occurs if you use the key as the subtraction symbol.
- # An error occurs if you input variable y in the function f(x). Variable x is treated as a variable. Other variables (A through Z, r, θ , excluding X and Y) are treated as constants and the value currently assigned to that variable is applied during the calculation.
- # An error occurs if you input variable x in the function g(y). Variable y is treated as a variable. Other variables (A through Z, r, θ , excluding X and Y) are treated as constants and the value currently assigned to that variable is applied during the calculation.

2. Differential Equations of the First Order

■ Separable Equation

Description

To solve a separable equation, simply input the equation and specify the initial values.

$$dy/dx = f(x)g(y)$$

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

- Press F1(1st) to display the menu of first order differential equations, and then select 1 (Separ).
- 3. Specify f(x) and g(y).
- 4. Specify the initial value for x_0 , y_0 .
- 5. Press F5 (SET) 1 (Param).
- 6. Specify the calculation range.
- 7. Specify the step size for h.
- Press F5 (SET) 2 (Output).
 Select the variable you want to graph, and then select a list for storage of the calculation results.
- 9. Make V-Window settings.
- 10. Press F6 (CALC) to solve the differential equation.



2-2 Differential Equations of the First Order

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Example

To graph the solutions of the separable equation $dy/dx = y^2 - 1$,

$$x_0 = 0, y_0 = \{0, 1\}, -5 \le x \le 5, h = 0.1.$$

Use the following V-Window settings.

$$Xmin = -6.2$$
, $Xmax = 6.2$, $Xscale = 1$

$$Ymin = -3.1$$
, $Ymax = 3.1$, $Yscale = 1$

Procedure

- 1) MENU DIFF EQ
- 2 F1(1st) 1 (Separ)
- (3) 1 EXE

 $ALPHA - (Y) \land 2 - 1 EXE$

4 0 EXE

SHIFT $\mathbf{X}(\{)\mathbf{0}$ $\mathbf{1}$ SHIFT $\div(\})$ EXE

- ⑤ F5 (SET) 1 (Param)
- 6 (-) 5 EXE

5 EXE

7 0 • 1 EXE ESC

- 8 F5 (SET) 2 (Output) F4 (INIT) ESC
- (9) [SHIFT] [OPTN] (V-Window)

(-) 6 • 2 EXE

6 • 2 EXE

1 EXE 🔻

(-) 3 • 1 EXE

3 • 1 EXE

1 EXE ESC

10 F6 (CALC)

Result Screen

(xo, yo) = (0,1) (xo, yo) = (0,0) TRACE[200MSKTCHG-SLU] [PICT]





2-3 Differential Equations of the First Order

■ Linear Equation

To solve a linear equation, simply input the equation and specify initial values.

$$dy/dx + f(x)y = g(x)$$

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

- 2. Press F1 (1st) to display the menu of differential equations of the first order, and then select 2 (Linear).
- 3. Specify f(x) and g(x).
- 4. Specify the initial value for x_0 , y_0 .
- 5. Press F5 (SET) 1 (Param).
- 6. Specify the calculation range.
- 7. Specify the step size for h.
- Press F5 (SET) 2 (Output).
 Select the variable you want to graph, and then select a list for storage of the calculation results.
- 9. Make V-Window settings.
- 10. Press [F6] (CALC) to solve the differential equation.



2-4 Differential Equations of the First Order

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Example

To graph the solution of the linear equation dy/dx + xy = x,

 $x_0 = 0, y_0 = -2, -5 \le x \le 5, h = 0.1.$

Use the following V-Window settings.

Xmin = -6.2, Xmax = 6.2, Xscale = 1

Ymin = -3.1, Ymax = 3.1, Yscale = 1

Procedure

1) MENU DIFF EQ

2 F1(1st) 2 (Linear)

(3) $[X,\theta,T]$ EXE

 X,θ,T EXE

4 0 EXE

(-) **2** EXE

⑤ **F5**(SET) **1** (Param)

6 (-) 5 EXE

5 EXE

7 0 • 1 EXE ESC

8 F5 (SET) 2 (Output) F4 (INIT) ESC

(9) [SHIFT] [OPTN] (V-Window)

(-) 6 • 2 EXE

6 • 2 EXE

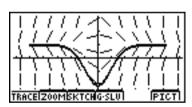
1 EXE 🔻

(-) 3 • 1 EXE

3 • 1 EXE

1 EXE ESC

10 F6 (CALC)





2-5 Differential Equations of the First Order

■ Bernoulli equation

To solve a Bernoulli equation, simply input the equation and specify the power of y and the initial values.

$$dy/dx + f(x)y = g(x)y^n$$

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

- 2. Press F1(1st) to display the menu of differential equations of the first order, and then select (3)(Bern).
- 3. Specify f(x), g(x), and n.
- 4. Specify the initial value for x_0 , y_0 .
- 5. Press F5 (SET) 1 (Param).
- 6. Specify the calculation range.
- 7. Specify the step size for h.
- Press F5 (SET) 2 (Output).
 Select the variable you want to graph, and then select a list for storage of the calculation results.
- 9. Make V-Window settings.
- 10. Press [F6] (CALC) to solve the differential equation.



2-6 Differential Equations of the First Order

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Example

To graph the solution of the Bernoulli equation $dy/dx - 2y = -y^2$, $x_0 = 0$, $y_0 = 1$, $-5 \le x \le 5$, h = 0.1.

Use the following V-Window settings.

Xmin = -6.2, Xmax = 6.2, Xscale = 1

Ymin = -3.1, Ymax = 3.1, Yscale = 1

Procedure

1) MENU DIFF EQ

2 F1(1st) 3 (Bern)

3 (-) 2 EXE

(-) 1 EXE

2 EXE

4 0 EXE

1 EXE

(5) **F5** (SET) 1 (Param)

6 (-) 5 EXE

5 EXE

7 0 • 1 EXE ESC

8 F5 (SET) 2 (Output) F4 (INIT) ESC

9 SHIFT OPTN (V-Window)

(-) 6 • 2 EXE

6 • 2 EXE

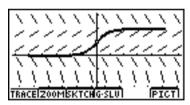
1 EXE 🔻

(-) 3 • 1 EXE

3 • 1 EXE

1 EXE ESC

10 F6 (CALC)





Others

To solve a general differential equation of the first order, simply input the equation and specify the initial values. Use the same procedures as those described above for typical differential equations of the first order.

$$dy/dx = f(x, y)$$

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

- 2. Press F1 (1st) to display the menu of differential equations of the first order, and then select 4 (Others).
- 3. Specify f(x, y).
- 4. Specify the initial value for x_0 , y_0 .
- 5. Press F5 (SET) 1 (Param).
- 6. Specify the calculation range.
- 7. Specify the step size for h.
- Press F5 (SET) (2 (Output).
 Select the variable you want to graph, and then select a list for storage of the calculation results.
- 9. Make V-Window settings.
- 10. Press F6 (CALC) to solve the differential equation.



2-8 Differential Equations of the First Order

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Example

To graph the solution of the first order differential equation $dy/dx = -\cos x$, $x_0 = 0$, $y_0 = 1$, $-5 \le x \le 5$, h = 0.1.

Use the following V-Window settings.

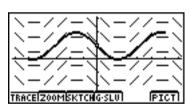
Xmin = -6.2, Xmax = 6.2, Xscale = 1

Ymin = -3.1, Ymax = 3.1, Yscale = 1

Procedure

- 1) MENU DIFF EQ
- ② [F1](1st) [4] (Others)
- 4 0 EXE
 - 1 EXE
- ⑤ **F5** (SET) **1** (Param)
- 6 (-) 5 EXE
 - 5 EXE
- 7 0 1 EXE ESC

- 8 F5 (SET) 2 (Output) F4 (INIT) ESC
- (9) [SHIFT] [OPTN] (V-Window)
 - (-) 6 2 EXE
 - 6 2 EXE
 - 1 EXE 🔻
 - (-) 3 1 EXE
 - 3 1 EXE
 - 1 EXE ESC
- (10) F6 (CALC)





3. Linear Differential Equations of the Second Order

Description

To solve a linear differential equation of the second order, simply input the equation and specify the initial values. Slope fields are not displayed for a linear differential equation of the second order.

$$y'' + f(x) y' + g(x)y = h(x)$$

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

- 2. Press [F2] (2nd).
- 3. Specify f(x), g(x), and h(x).
- 4. Specify the initial value for x_0 , y_0 , y'_0 .
- 5. Press F5 (SET) 1 (Param).
- 6. Specify the calculation range.
- 7. Specify the step size for h.
- Press F5 (SET) 2 (Output).
 Select the variable you want to graph, and then select a list for storage of the calculation results.
- 9. Make V-Window settings.
- 10. Press F6 (CALC) to solve the differential equation.



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Example

To graph the solution of the linear differential equation of the second order $y'' + 9y = \sin 3x$, $x_0 = 0$, $y_0 = 1$, $y'_0 = 1$, $0 \le x \le 10$, h = 0.1.

Use the following V-Window settings.

Xmin = -1, Xmax = 11, Xscale = 1

Ymin = -3.1, Ymax = 3.1, Yscale = 1

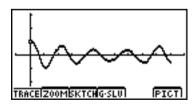
Procedure

- 1) MENU DIFF EQ
- ② F2 (2nd)
- (3) (0) EXE
 - 9 EXE
 - sin 3 $[X,\theta,T]$ EXE
- 4 0 EXE
 - 1 EXE
 - 1 EXE
- (5) [F5] (SET) 1 (Param)
- 6 0 EXE
 - 1 0 EXE
- 7 0 1 EXE*1ESC
 - *1 Parameter
 Xrange
 min :0
 max :10
 h :0.1
 Step :1
 SF :12

- 8 F5 (SET) 2 (Output) F4 (INIT) ESC
- 9 SHIFT OPTN (V-Window)
 - (-) 1 EXE
 - 1 1 EXE

 - (-) 3 1 EXE
 - 3 1 EXE
 - 1 EXE *2 ESC
- 10 F6 (CALC)







4. Differential Equations of the Nth Order

You can solve differential equations of the first through ninth order. The number of initial values required to solve the differential equation depends on its order.

• Enter dependent variables $y, y', y'', y''^{(3)}, \dots, y^{(9)}$ as follows.

■ Differential Equation of the Fourth Order

The following example shows how to solve a differential equation of the fourth order.

$$y^{(4)} = f(x, y, \dots, y^{(3)})$$

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

- 2. Press [F3] (N-th).
- 3. Press [3](n) 4 to select a differential equation of the fourth order.
- 4. Specify *y*⁽⁴⁾.
- 5. Specify the initial value for x_0 , y_0 , y'_0 , y''_0 , and $y^{(3)}_0$.
- 6. Press F5 (SET) 1 (Param).
- 7. Specify the calculation range.
- 8. Specify the step size for h.
- Press F5 (SET) 2 (Output).
 Select the variable you want to graph, and then select a list for storage of the calculation results.
- 10. Make V-Window settings.
- 11. Press F6 (CALC) to solve the differential equation.

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Example

To graph the solution of the differential equation of the fourth order below

$$y^{(4)} = 0$$
, $x_0 = 0$, $y_0 = 0$, $y'_0 = -2$, $y''_0 = 0$, $y^{(3)}_0 = 3$, $-5 \le x \le 5$, $h = 0.1$.

Use the following V-Window settings.

Xmin = -6.2, Xmax = 6.2, Xscale = 1

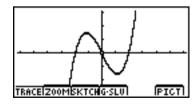
Ymin = -3.1, Ymax = 3.1, Yscale = 1

Procedure

- 1 MENU DIFF EQ
- ② F3(N-th)
- 3 F3(n) 4 EXE
- 4 0 EXE
- (5) (0) [EXE]
 - 0 EXE
 - (-) 2 EXE
 - O EXE
 - 3 EXE
- ⑥ F5 (SET) 1 (Param)
- 7 (-) 5 EXE
 - 5 EXE
- 8 0 1 EXE*1ESC
 - *1 Parameter
 Xrange
 min :-5
 max :5
 h :0.1
 Step :12
 TNIT

- 9 F5 (SET) 2 (Output) F4 (INIT) ESC
- (V-Window)
 - (-) 6 2 EXE
 - 6 2 EXE
 - 1 EXE 🔻
 - (-) 3 1 EXE
 - 3 1 EXE
 - 1 EXE *2 ESC
- 11) F6 (CALC)







■ Converting a High-order Differential Equation to a System of the First Order Differential Equations

You can convert a single N-th order differential equation to a system of n first order differential equations.

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

Execution (N = 3)

- 2. Press [F3] (N-th).
- 3. Press [F3](n) 3 to select a differential equation of the third order.
- 4. Perform substitutions as follows.

$$y' \rightarrow Y1 (F3(y_{(n)})1)$$

$$y'' \rightarrow Y2 (F3(y_{(n)})2)$$

$$y^{(3)} \to Y3 (F3(y_{(n)})3)$$

- 5. Specify the initial value for x_0 , y_0 , y'_0 , and y''_0 .
- 6. Press **F2**(→SYS).
- 7. Press EXE (Yes).
- The entered differential equation is converted to a system of three first order differential equations. Initial values are also converted accordingly.



Example

Express the differential equation below as a set of first order differential equations.

$$y^{(3)} = \sin x - y' - y'', x_0 = 0, y_0 = 0, y'_0 = 1, y''_0 = 0.$$

Procedure

- 1) MENU DIFF EQ
- ② [F3] (N-th)
- $\mathfrak{F}\mathfrak{J}(n)\mathfrak{I}$
- 4 $\sin (x,\theta,T) F3(y(n))$ 1 F3(y(n)) 2 EXE
- (5) O EXE
 - O EXE
 - 1 EXE
 - O EXE
- ⑥ F2 (→SYS)
- 7 EXE (Yes)

The differential equation is converted to a set of first order differential equations as shown below.

$$(y_1)' = dy/dx = (y_2)$$

$$(y_2)' = d^2y/dx^2 = (y_3)$$

$$(y_3)' = \sin x - (y_2) - (y_3).$$

Initial values are also converted to $(x_0 = 0)$, $((y_1)_0 = 0)$, $((y_2)_0 = 1)$, and $((y_3)_0 = 0)$).

Result Screen



On the system of first order differential equations screen, dependent valuables are expressed as follows.

$$(y_1) \rightarrow Y1$$

$$(y_2) \rightarrow Y2$$

$$(y_3) \rightarrow Y3$$

5. System of First Order Differential Equations

A system of first order differential equations, for example, has dependent variables (y_1) , (y_2) ,, and (y_9) , and independent variable x. The example below shows a system of first order differential equations.

$$(y_1)' = (y_2)$$

 $(y_2)' = -(y_1) + \sin x$

Set Up

1. From the Main Menu, enter the DIFF EQ Mode.

Execution

- 2. Press F4 (SYS).
- 3. Enter the number of unknowns.
- 4. Enter the expression as shown below.

$$(y_1) \to Y1 (F3(y_n)1)$$

 $(y_2) \to Y2 (F3(y_n)2)$
:

 $(y_9) \rightarrow Y9 (F3(y_n)9)$

- 5. Specify the initial value for x_0 , $(y_1)_0$, $(y_2)_0$ and so on, if necessary.
- 6. Press F5 (SET) 1 (Param).
- 7. Specify the calculation range.
- 8. Specify the step size for h.
- Press F5 (SET) 2 (Output).
 Select the variable you want to graph, and then select a list for storage of the calculation results.
- 10. Make V-Window settings.
- 11. Press \digamma 6 (CALC) to solve the system of first order equations for y_1 , y_2 , and so on.



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Example 1 To graph the solution of first order differential equations with two unknowns below.

$$(y_1)'=(y_2), (y_2)'=-(y_1)+\sin x, x_0=0, (y_1)_0=1, (y_2)_0=0.1, -2 \le x \le 5, h=0.1.$$

Use the following V-Window settings.

Xmin = -3, Xmax = 6, Xscale = 1

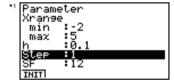
Ymin = -2. Ymax = 2. Yscale = 1

Procedure

- 1) MENU DIFF EQ
- 2 F4(SYS)
- ③ **F2**(2)
- 4 F3 (yn) 2 EXE

 \bigcirc F3 (yn) 1 \bigcirc Sin (X,θ,\overline{I}) EXE

- (5) (0) [EXE]
 - 1 EXE
 - 0 1 EXE
- 6 F5 (SET) 1 (Param)
- 7 (-) 2 EXE
 - 5 EXE
- 8 0 1 EXE*1 ESC



- 9 F5 (SET) 2 (Output) F4 (INIT)
 - **▼ F1**(SEL)

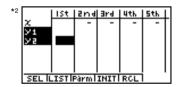
(Select (y_1) and (y_2) to graph)*2

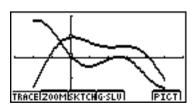
ESC

(10) SHIFT (OPTN) (V-Window)

(-) **3** EXE

- 6 EXE
- (-) 2 EXE
- 2 EXE
- 1 EXE ESC
- 11) F6 (CALC)







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Example 2 To graph the solution of the system of first order differential equations below.

$$(y_1)' = (2 - (y_2)) (y_1)$$

 $(y_2)' = (2 (y_1) - 3) (y_2)$

$$x_0 = 0$$
, $(y_1)_0 = 1$, $(y_2)_0 = 1/4$, $0 \le x \le 10$, $h = 0.1$.

Use the following V-Window settings.

Xmin = -1, Xmax = 11, Xscale = 1

Ymin = -1, Ymax = 8, Yscale = 1

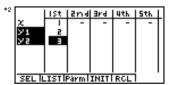
Procedure

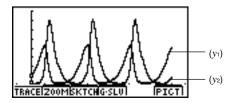
- 1) MENU DIFF EQ
- 2 F4(SYS)
- ③ **F2**(2)
- (4) (2 = F3(yn)2) X F3(yn)
 - 1 EXE
 - $(2 \times F3(yn)1 3)$
 -) **X** F3 (yn) **2** EXE
- (5) (0) [EXE]
 - 1 EXE
 - 1 : 4 EXE
- ⑥ F5 (SET) 1 (Param)
- (7) (0) EXE
 - 1 0 EXE
- 8 0 1 EXE*1ESC
 - *1 Parameter
 Xrange
 min :0
 max :10
 h :0.1
 Step :12
 TNTT

- 9 F5 (SET) 2 (Output) F4 (INIT)
 - ▼ F1 (SEL)

(Select (y_1) and (y_2) to graph.)

- \triangle \bigcirc F2 (LIST) 1 EXE (Select LIST1 to store the values for x in LIST1)
- **F2** (LIST) **2 EXE** (Select LIST2 to store the values for (y_1) in LIST2)
- \bigcirc F2(LIST) 3 EXE (Select LIST3 to store the values for (y_2) in LIST3)*2
- ESC
- 10 SHIFT OPTN (V-Window)
 - (-) 1 EXE 1 1 EXE 1 EXE •
 - (-) 1 EXE 8 EXE 1 EXE ESC
- 11 F6 (CALC)







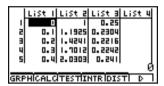
5-4 System of First Order Differential Equations

■ Further Analysis

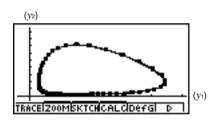
To further analyze the result, we can graph the relation between (y_1) and (y_2) .

Procedure

- (1) (MENU) STAT
- ② List 1, List 2, and List 3 contain values for x, (y_1) , and (y_2) , respectively.
- 3 F1(GRPH) 5 (Set)
- 4 F1(GPH1)
- \bigcirc **F2**(xy)
- ⑥ **(F1)** (LIST) **2 (XLIST = LIST2:** (y_1))
- \bigcirc FI (LIST) 3 EX (YLIST = LIST3: (y_2))
- 8 F1 (GRPH) 1 (S-Gph1)



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Important!

- This calculator may abort calculation part way through when an overflow occurs part way
 through the calculation when calculated solutions cause the solution curve to extend into
 a discontinuous region, when a calculated value is clearly false, etc.
- The following steps are recommended when the calculator aborts a calculation as described above.
 - If you are able to determine beforehand the point where the solution curve overflows, stop the calculation before the point is reached.
 - 2. If you are able to determine beforehand the point where the solution curve extends into a discontinuous region, stop the calculation before the point is reached.
- 3. In other cases, reduce the size of the calculation range and the value of h (step size) and try again.
- 4. When you need to perform a calculation using a very wide calculation range, store intermediate results in a list and perform a new calculation starting from step 3 using the stored results as initial values. You can repeat this step multiple times, if necessary.

■ SET UP Items

G-Mem $\{G-Mem\ 20\}/\{1-20\}....$ Specifies a memory location $\{G-Mem\ No.\}$ for storage of the latest graph functions.

Note the following regarding SET UP screen settings whenever using the DIFF EQ Mode.

The DIFF EQ Mode temporarily stores data into Graph Memory whenever a differential equation calculation is performed. Before the calculation, DIFF EQ stores the latest graph functions into the currently specified Graph Memory (G-Mem) location. After the calculation, it recalls the graph functions from the specified G-Mem location, without deleting the G-Mem data. Because of this, you should specify the G-Mem location (number) where the DIFF EQ Mode stores the graph functions.

